Problem of the Month
Party Time

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:
In the Problem of the Month *Party Time,* students use mathematical concepts of logic, reasoning, and counting methods. The mathematical topics that underlie this POM are knowledge of logic, deductive reasoning, counting principles/strategies, and a variety of mathematical representations such as tree diagrams, Venn diagrams, tables, charts, and matrices.

In the first level of the POM, students determine the number of guests invited to a party through examination of set invites, guest inviting other sets of guests. In part B, students are asked to determine the number of girls with short red hair at a party given a number of logic clues. The students need to partition a whole using simple fraction (1/2 and 1/4). The students apply logical reasoning to determine the number of red-headed girls at the party. In level C, students are presented with a set of clues regarding the names, costumes, and time of arrival at the party. The students are asked to match each of the partygoers to their names, costumes and arrival times. In level D, the students are asked to determine when a game is fair for both players. Students justify their findings and explain when the game is fair. In the final level, students are asked to solve a complex logic puzzle. Students must defend their solution and explain how they solved the puzzle.

Problem of the Month
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Level A

Cindy had a party. She invited two guests. Her guests each invited four guests, and then those guests each invited three guests.

How many people were at Cindy's party?

Explain how you determined your solution.

Level B

At Leslie's party ¼ of the people had long hair. One half of the people at the party were boys, ¼ of the girls had short blond hair. None of the boys had long hair.

If there were 32 guests, what is the maximum number of girls who could have had short red hair?

Show how you determined you answer and why you know you have a correct solution.

Level C

Mia, Jake, Carol, Barbara, Ford and Jeff are all going to a costume party. Figure out which person is wearing what costume and when they arrived at the party.

- The person that arrived fourth was wearing bathing suit.
- Barbara was the last to arrive.
- Jake and Mia arrived and stayed together.
- The first person was dressed as a French Maid.
- Superman arrived right before Barbara.
- The Potato Heads were always together at the party.
- Ford was a Surfer Dude.
- The French Maid was not Carol.
- The Vampire arrived after Superman.

Level D

Your Aunt is having a baby. You have created a party game for a baby shower. It is called pick the gender. You put pink and blue tiles into a bag. You ask two guests to pick one tile out of the bag without looking. You tell your guests that if they are the same color, player A wins and if they are two different colors, then player B wins.

How many tiles of which colors did you put into the bag to make sure that both players have an equal chance of winning?

Explain your solution and why it is fair.

Level E

A man and his wife invite 5 other couples to a dinner party. As the guests arrive for drinks before dinner, they shake hands. Not everybody shakes everybody's hands, and of course no one shakes hands with his own spouse. Later, as they sit down to dinner, the host asks each other person, including his wife, "how many hands you shake?" He notices, to his surprise, that each respondent shook a different number of hands.

How many did his wife shake?

Explain your solution and justify your reasoning.

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Primary Version Level A

Materials: Sets of counters.

Discussion on the rug: (Teacher asks the class.) "Who likes to have parties in our class? We are going to solve a problem that is about inviting friends to a party. Who would like to be our party host?" (Teacher invites a student to come forward). The teacher says to the host, "Let's start by inviting three friends to the party." (The student picks three friends to come forward.) "How many people are at the party?" the teacher asks the class. (Students share their answers and explain how they know.) The teacher says, "Suppose each of the friends phone two people to come to the party, how many will be at the party altogether?" (Students share their ideas and discuss solutions. Then they actually act it out and count the total number of people at the party).

In small groups: (Students have counters available.)
Teacher says, "Cindy had a party. She invited two guests. Her guests each invited four guests, and then those guests each invited three guests. How many people were at Cindy's party?" (Students work together to find a solution. After the students are done, the teacher asks students to share their answers and method.)

At the end of the investigation: (Students either discuss or dictate a response to this summary question.) "Explain and show how you know how many people are at the party."

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Task Description – Level A

This task challenges a student to use logic and counting principles to find the number of guests at a party if guests invite guests at a given ratio.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

3.OA.1 Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, identify the outcomes in the sample space, which compose the event.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are

comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Party Time

Task Description – Level B

This task challenges a student to use logic and fractions to find a part of a set that meets several constraints. Students must use fractional parts of the set to help calculate their answers. Students need to find a systematic way to organize their results as they consider each constraint.

Common Core State Standards Math - Content Standards

Number and Operations - Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication and to multiply a fraction or whole number by a fraction.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

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Problem of the Month

Party Time

Task Description – Level C

This task challenges a student to use logic and logic strategies such as tables, charts, matrices to solve a problem with a series of clues about guests, their costumes, and their arrival time. Students need to make sense of multiple constraints to find a solution and check the solution against all the constraints.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, identify the outcomes in the sample space, which compose the event.

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Problem of the Month

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Task Description – Level D

This task challenges a student to reason about a fair game in the context of picking tiles of different colors from a bag. Students must define a sample space that will give an equal probability of getting matching or nonmatching colors when 2 tiles are drawn from the bag without replacement.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an even that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model of observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, identify the outcomes in the sample space, which compose the event.

High School – Statistics and Probability – Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").

S-CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.8 apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B/A)=P(B)P(A/B), and interpret the answer in terms of the model.

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Problem of the Month

Party Time

Task Description – Level E

This task challenges a student to think about a logic problem about shaking hands with each new set of arriving guests. However there are multiple constraints, such as at the end each person must have a different number of total handshakes. Students must defend their solution and explain how they solved the puzzle.

Common Core State Standards Math - Content Standards

Statistics and Probability

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7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, identify the outcomes in the sample space, which compose the event.

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Task Description – Primary Level

This task challenges a student to use logic and counting principles to find the number of guests at a party if guests invite guests at a given ratio. Students use people and objects to help make the mathematics accessible.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20 count out that many objects.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations verbal explanations, expressions or equations.

Represent and solve problems involving addition and subtraction.

1.OA.2 Solve word problems that call for addition of three whole numbers, whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

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